

# GROUND VIBRATIONS NEAR EXPLOSIONS.\*

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## INTRODUCTION

ONE WOULD expect that, since seismic waves from explosions are the basis of a whole industry (seismic surveying), their nature would have been thoroughly investigated and described. However, the exploration geophysicist is not primarily interested in the nature of the seismic pulses, but in their velocities and the paths they travel to his recording instruments. It is common practice in studying seismic exploration records to assume that only compressional pulses are clearly recorded, though occasionally the presence of some transverse wave energy is postulated to explain otherwise incomprehensible observations. Other types of wave motion are treated as part of the background noise, and wherever possible are excluded from the recorded spectrum by the use of appropriate filters in the amplifiers.

However, proper interpretation of the records obtained requires some knowledge of the detailed character of the ground motion; and, therefore, any information on the nature of the seismic energy which is generated by an explosion is certain to have some value. The series of experiments described here was undertaken with the intention of increasing the knowledge of the basic seismic forms to be expected in the record of an explosion.

## PREVIOUS EXPERIMENTAL WORK

Most of the work done in the past has been on large explosions recorded at near-by permanent seismic observatories. These investigations treat primarily data recorded many kilometers from the source of the energy. Because of the large distances, the shapes of the pulses are much altered from what they were near their source, and are not well suited to studies of their detailed nature.

The form of a compressional pulse near an explosion was studied by J. A. Sharpe (1942).<sup>1</sup> He analyzes the pulse shape to be expected from theoretical considerations and compares it with those observed. Since the initiating forces in the ideal case are entirely radial, there should be no shear waves directly generated, a conclusion borne out by observation for explosions buried at sufficient depth beneath the surface. Shear waves are, however, to be expected from a superficial explosion where force is applied vertically to the surface (Lamb, 1904).

In practice we do not encounter an ideal homogeneous, isotropic, elastic medium. Although the deeper layers of rock may be considered homogeneous,

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<sup>1</sup> All references are to the bibliography at the end of the paper.

the surface layer certainly is not, and can be satisfactorily approximated only by assuming that it varies at least in the vertical direction. Isotropy, also, is only a first approximation; foliated and bedded rocks have different properties perpendicular and parallel to the foliation and bedding. Study of the transmission of body waves through the earth indicates that at large depths the condition of elasticity is approximately held for seismic waves. The transmission of compressional waves through a medium of the type of the uppermost layer has been studied by Lampson and his co-workers (Lamp-

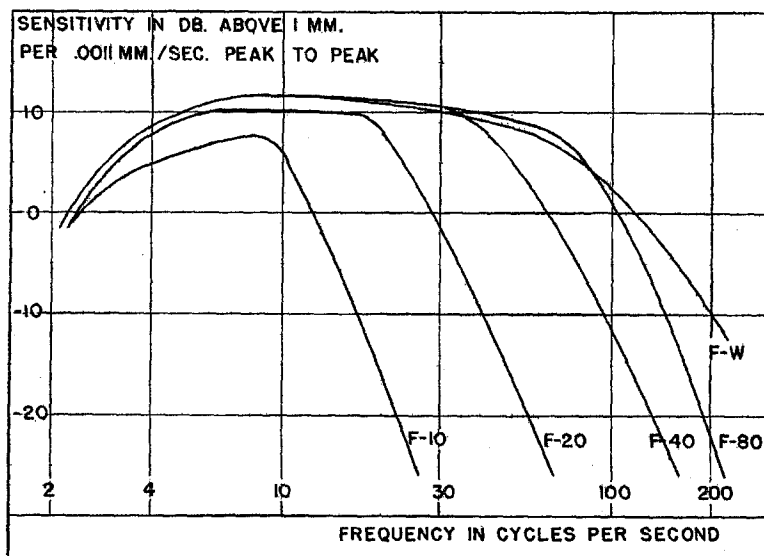


Fig. 1. Calibration curves of vertical component of seismograph.

son, 1942, 1946; Rust and Mounce, 1942; Weatherby, 1943); but a great deal more experimental work will have to be done before the wave mechanics in such a medium can be described quantitatively for all cases. Yet most of our observations of seismic waves are made at the upper boundary of a layer of this sort.

A direct approach to the problem was made by L. D. Leet (1939, 1946; also Leet and Ewing, 1932). Leet recognized two types of waves in records from explosions besides the well-known compressional, transverse, Love, and Rayleigh waves. The first of these, the "C" or "coupled" wave, is characterized by simultaneous in-phase motion on all three components. The second, the "H" or "hydrodynamic" wave, is similar to the Rayleigh wave, except that whereas the particle motion in a Rayleigh wave is retrograde with respect to the direction of propagation, in the hydrodynamic wave the particle motion is direct, being like that of a gravity wave in a liquid. H has been reported until now

only in the atomic bomb record made in the course of the Los Alamos test and in the record of one explosion in New England. C was also observed in these records, and in other records from blasts in New England. Pulses which resemble C and H were observed in this investigation.

### THE APPARATUS

The apparatus used in this research was generously lent by the United Geophysical Company. It consisted of three electromagnetic induction seismometers, three amplifiers, and a recording oscillograph designed for refraction-type geophysical prospecting. The three seismometers were originally intended to respond only to the vertical component of ground motion. Two of them were modified to respond, instead, to horizontal components, one toward and away from the shot, the other perpendicular to this. Figure 1 is the calibration curve of the vertical component for sinusoidal motion. The curves for the other two components are similar. The low-frequency limit was determined by the sensitivity of the seismometers, which decreased rapidly below their natural frequency of  $3\frac{1}{2}$  cycles per second (henceforth abbreviated c.p.s.). The high-frequency limit could be adjusted by means of a filter. The sensitivity of the horizontal components to vertical motion was about  $1/10$  that to horizontal motion. The sensitivity of the vertical component to horizontal motion was about  $1/50$  that to vertical motion.

The records taken in the field were naturally not sinusoidal (figs. 3-5), and hence the amplitudes of the ground motions could not be determined precisely from the calibration curves. In addition, much of the measured motion had periods of more than  $\frac{1}{4}$  second. Below 5 c.p.s. the gain varies with frequency, and an appreciable amount of error is introduced by computing the amplitude of motion for an incorrect frequency.

### LOCATION OF THE SURVEY AND FIELD PROCEDURE

The records were made about 42 kilometers southeast of Los Angeles (see map, fig. 2). Measurements were made at 14 locations along an east-west line 97 to 3,284 meters from the shot point. The seismometers were set on the surface, or half buried in the ground at the side of a road, or in a dry irrigation ditch about a meter deep. The recording apparatus was mounted in a truck. The charges, consisting of  $2\frac{1}{2}$  or 5 pounds of Hercules 60 per cent *petrogel*, were set off at depths ranging from 6.7 to 12.5 meters.

Little detailed information is available on the geology of the district. The surface material is everywhere a fine-grained clayey soil. The surface rocks are thought to be Recent alluvium, probably overlying Pleistocene marine terraces and alluvial fan deposits of considerable thickness. The water table is about three meters beneath the surface.

Study of the refracted compressional waves indicates that there is a "weath-

ered" or "low-velocity" layer between 1.2 and 8.4 meters thick the compressional wave velocity of which is between 120 and 735 m/s., probably increasing rapidly with depth, a second layer between 29 and 106 meters thick whose compressional wave velocity is 1,565 m/sec., and a third layer of unknown thickness the compressional wave velocity of which is 1,950 m/sec.

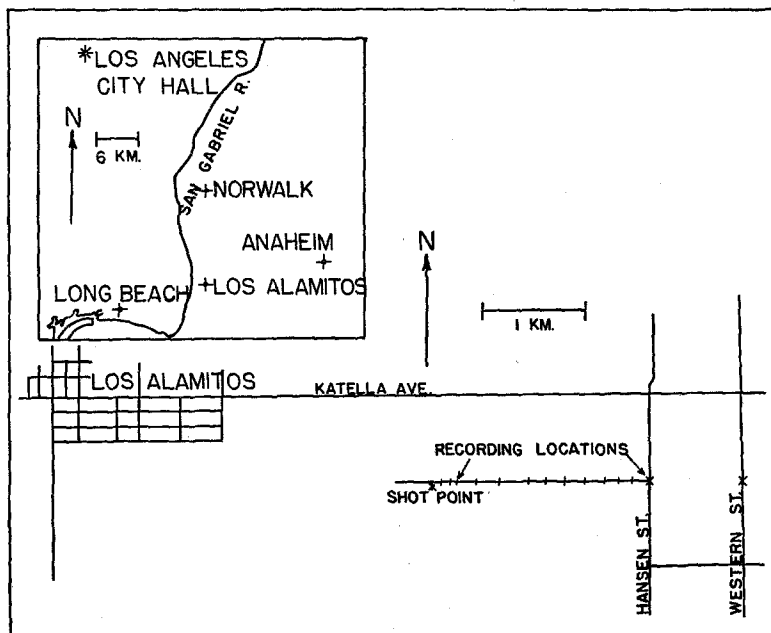


Fig. 2. Location where measurements were made.

### THE RECORDS

Figures 3-5 are tracings of a selected group of the records. In each case the three traces represent, from top to bottom, vertical, longitudinal, and transverse motion. The sensitivities of the three channels are not identical; the relative amplifications of the horizontal channels with respect to the vertical in decibels at 10 c.p.s. are given beside each horizontal trace. Positive values correspond to greater amplification in the horizontal channel, negative to lesser.

On the charts there is also shown the amplification of the vertical channel at 10 c.p.s. in decibels above 1 mm. peak-to-peak spot deflection on the paper per millimeter per second peak ground velocity; also, the nominal cut-off frequency of the filter, the distance from the shot to the recording station, the size of the charge, and the depth at which it was detonated. Timing marks are given representing half-second intervals following the shot.

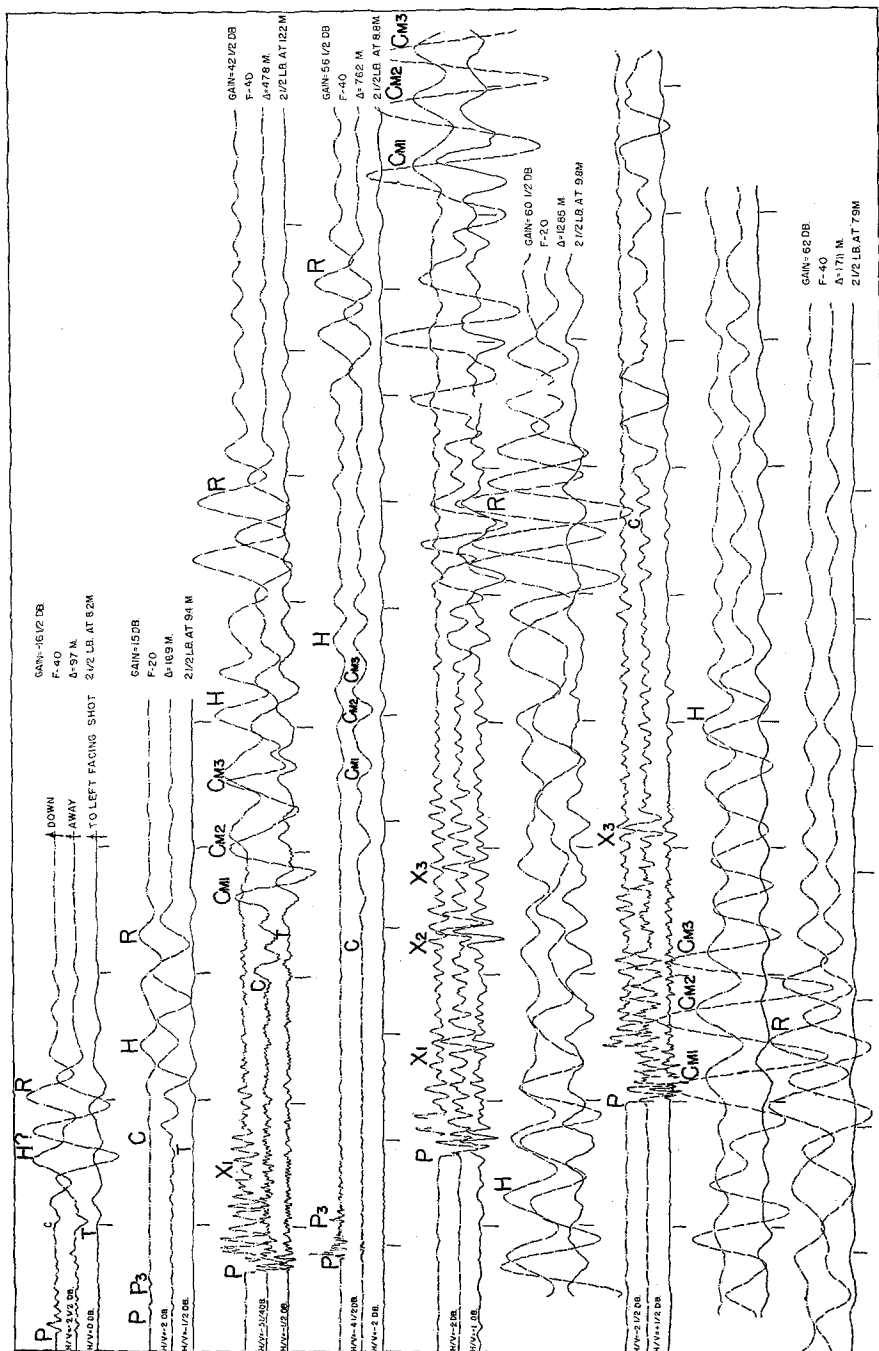


Fig. 3. Records of ground motion at distances out to 1,171 meters. P is the first arrival;  $P_3$ ,  $X_1$ ,  $X_2$  and  $X_3$  are later body wave pulses; C is the beginning of the C pulse;  $C_{m1}$ ,  $C_{m2}$  and  $C_{m3}$  are a characteristic group of three maxima of C; T is the beginning of long-period motion on the transverse component; H is the maximum of the H pulse; and R is the maximum of R.

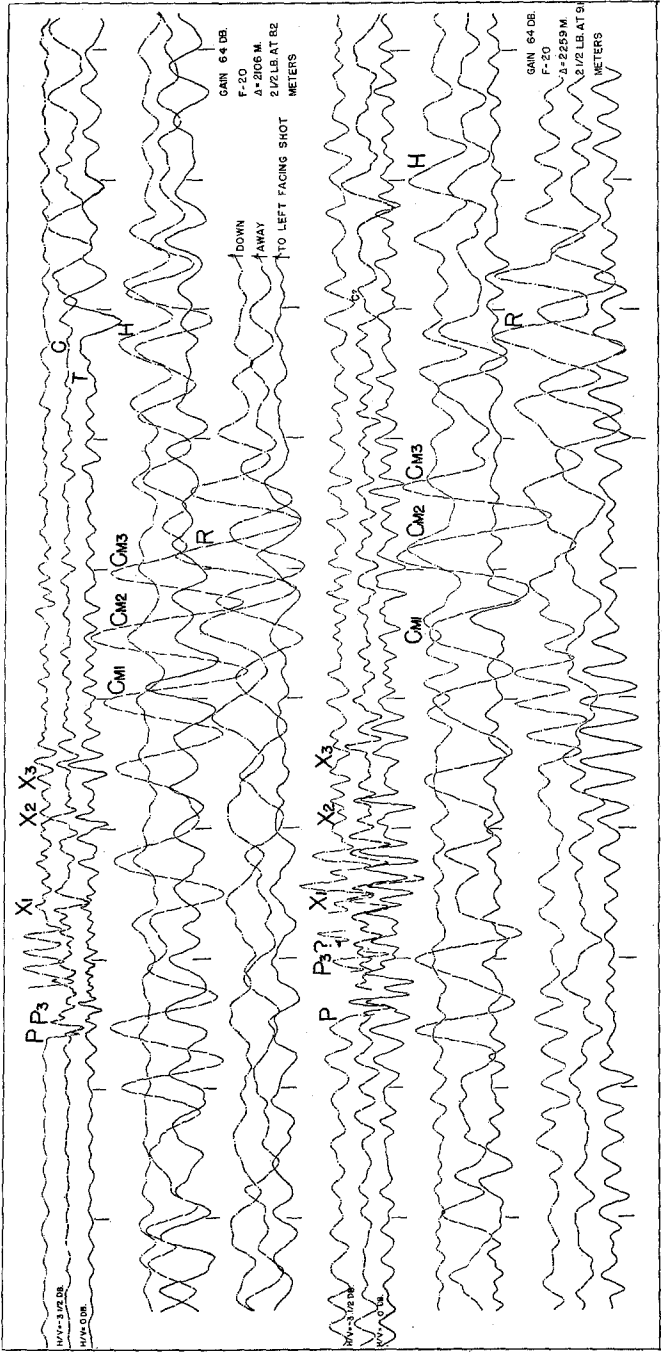


Fig. 4. Records of ground motion at 2,106 and 2,259 meters. For explanation of symbols see figure 3.

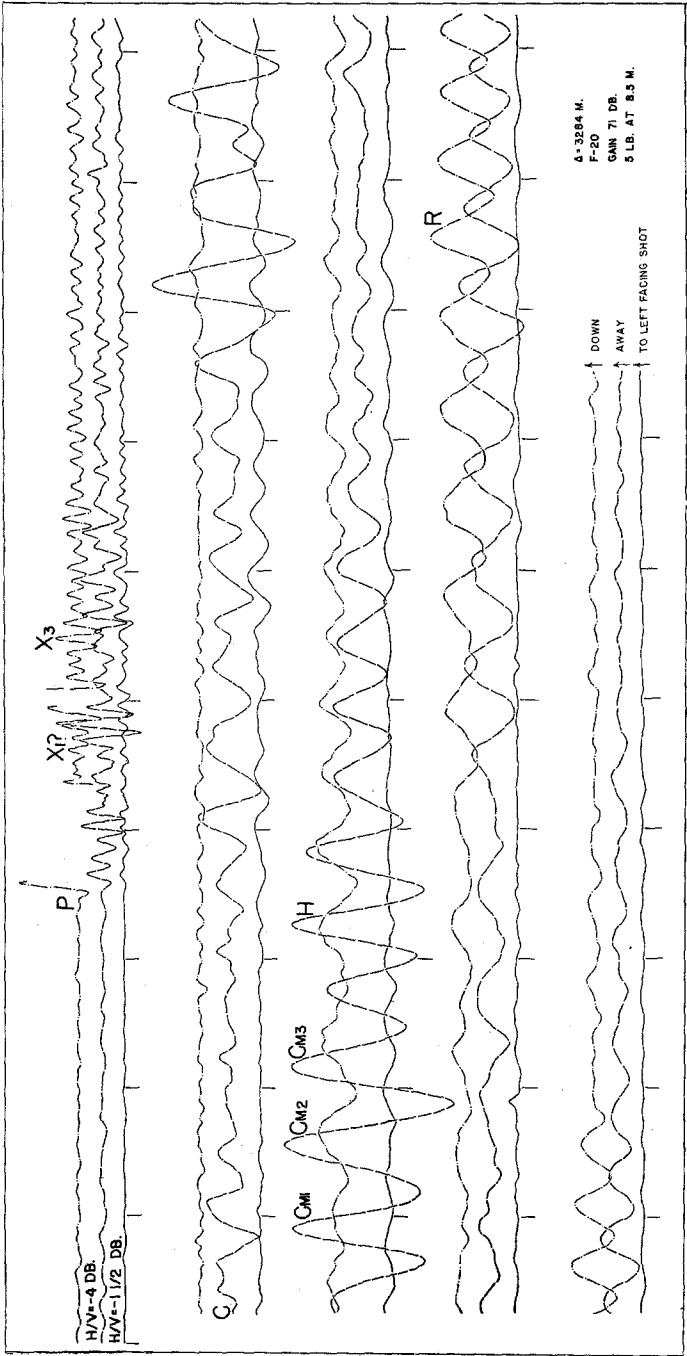


Fig. 5. Record of ground motion at 3,284 meters. For explanation of symbols see figure 3.





## PRINCIPAL BODY WAVES

Where the author has succeeded in correlating the separate pulses, a representative symbol is given above the beginning of that pulse. "P" is the first wave to arrive at the recording station, regardless of what path it traveled by. It arrives at the surface at angles of  $28^\circ$  or less with the vertical, this angle in general decreasing as the distance from the shot increases.

Following P on the records are a number of more obscure pulses. Four which were correlated on most of the records are marked on the travel-time curves, figure 6, together with a few of the more prominent noncorrelatable pulses. These four are also marked on figures 3-5. Unfortunately, the beginnings of the pulses are not distinct enough to be used for detailed calculations of sub-surface relations.

The first of the four pulses is labeled "P<sub>3</sub>." It appears to be similar to P in the nature of its motion. It is probably a refracted compressional pulse which has penetrated deeper than P. "X<sub>1</sub>," "X<sub>2</sub>," and "X<sub>3</sub>," which follow, can be recognized on all three components, but usually not clearly on all on any one record. X<sub>1</sub> is often a prolonged oscillation; it may be a combination of several pulses. It commonly has a strong vertical component, whereas X<sub>2</sub> and X<sub>3</sub> are most easily recognized on the horizontal components. They are probably body waves penetrating beneath the third layer. One or more of them may be shear waves.

## DISPERSED WAVES ON THE LONGITUDINAL COMPONENT

The next group of waves to arrive, which starts with the emergent pulse marked "C" on the records, consists predominantly of longitudinal motion (fig. 7). There is some correlated motion on the vertical component at short distances.

The motion illustrated by figure 7, and the similar diagrams which follow, are given in terms of velocity, not displacement. They differ from true particle motion diagrams, owing to relative phase shift between different frequency components and a frequency factor in the expressions for the amplitudes. Since the pulses recorded are the sums of many frequency components, the shape of the particle path will be different from that shown in the figures, although they do indicate the nature and direction of the particle motion during the passage of the pulse.

The beginning of C is recognizable only by a gradual growth in size of the motion. In general the average amplitude increases throughout the time of arrival of this pulse, building up to a series of maxima, with smaller velocities in between. These maxima can sometimes be correlated on records taken at successive distances, but never continuously over long distances. A number of such correlations are shown in figure 6. Their pattern indicates a group ve-

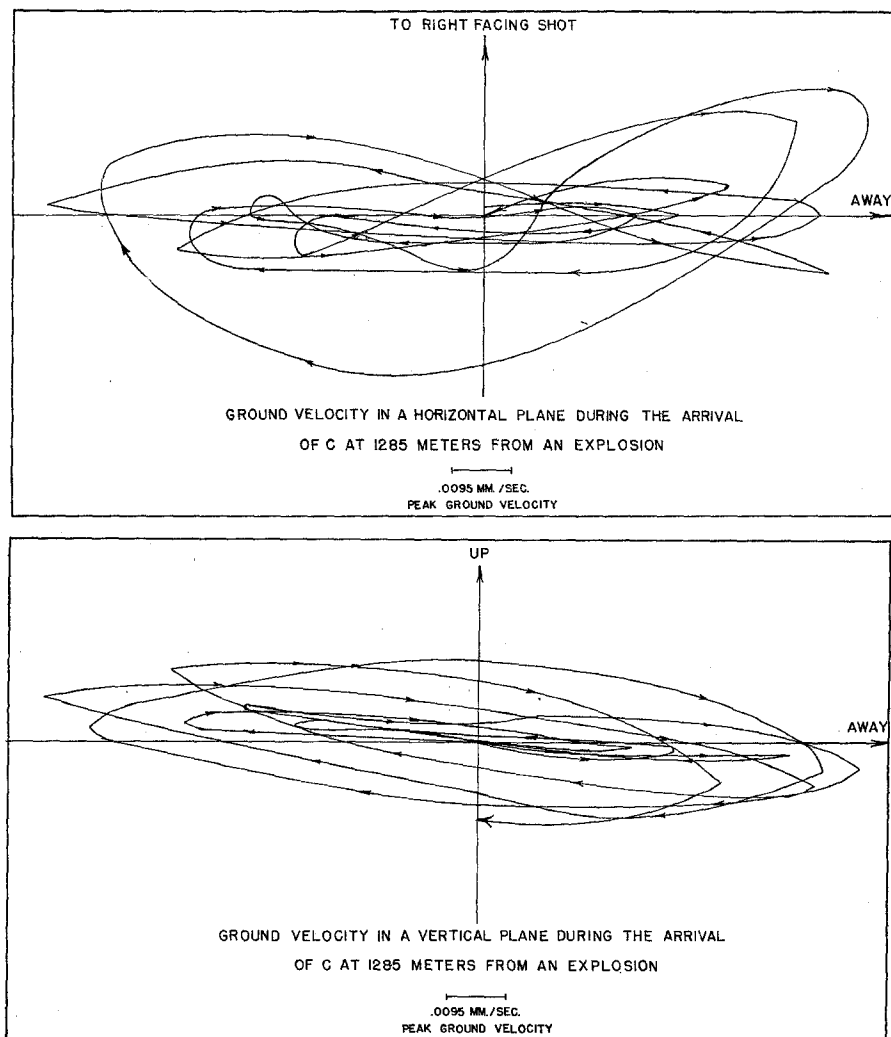


Fig. 7. Graph of ground velocity during the arrival of C.

locity, showing that C is dispersed. The times of arrival of all the outstanding maximum away-from-the-shot velocities of C have been plotted. The last C motion commonly includes a series of strikingly large maxima marked  $C_{M1}$ ,  $C_{M2}$ , and  $C_{M3}$  on figures 3-5 and CM on figure 6. The maximum amplitude shifts to later and later curves as the distance from the source of the waves increases.

A line can be roughly drawn which represents the last recognized C motion, just as the emergent pulse labeled C on the records corresponds to the observed maximum velocity phase. However, the suddenness of the cessation is not

certain since a different type of motion begins prior to the end of C. During the arrival of C the period gradually increases. The last C waves on the records have periods of .22 to .31 seconds. This is approximately the cut-off frequency of the recording system, which may explain the abrupt decrease in amplitude at this point. On the other hand, sudden termination of motion would suggest a minimum group velocity, as discussed by Jeffreys (1925) and Stoneley (1925) for earthquakes.

An interesting feature of C is that, like P, it seems to be refracted. Its travel-time curve appears to consist of two or three straight lines for both the maximum and minimum velocities.

The C pulses resemble Leet's (1939, 1946) "coupled" waves. They occur at the same place with respect to other types of motion on both Leet's records and those shown here. However, in the Los Alamitos region the motion was confined almost entirely to the longitudinal component, whereas Leet recorded considerable motion on all three components, though strongest on the longitudinal.

The motion of the ground during the passage of C is not that of any of the well-known wave types of seismology. Seismic body waves show no appreciable dispersion, and surface waves would not be refracted. Either C is a new type of wave, or the ground at Los Alamitos is so different from that assumed by classical theory that one of the well-known body or surface waves is thereby modified to the motion observed. Since the C waves are dispersed, the suggestion is strong that they are surface waves. But since the motion is largely restricted to the longitudinal component, it resembles neither a Love nor a Rayleigh wave. Fu (1947) has considered the waves resulting from the incidence of spherically symmetrical compressional waves on an interface, and concluded that besides body and Rayleigh waves there result several other types, including a compressional surface wave traveling with the velocity of the body shear wave and falling off in amplitude as the square of the distance from the source of the energy. If C were of this nature, its absorption coefficient would be given by the expression:

$$\alpha = \frac{1}{\Delta_2 - \Delta_1} \log_e \frac{A_1 \Delta_1^2}{A_2 \Delta_2^2} \quad (1)$$

where  $A_n$  is the amplitude of the vibrational velocity at distance  $\Delta_n$  from the source of the waves. The attenuation constant determined for the Los Alamitos region using the observed  $A_n$  and  $\Delta_n$  is negative, an impossibility. Therefore, C cannot be a wave of the type described by Fu.

If C were an ordinary surface wave, equation (1) would become:

$$\alpha = \frac{1}{\Delta_2 - \Delta_1} \log_e \frac{A_1 \Delta_1^{\frac{3}{2}}}{A_2 \Delta_2^{\frac{3}{2}}} \quad (2)$$

The corresponding  $\alpha$  in this case is approximately .0012 per meter.

Since the ground motion is largely longitudinal, C could be caused by body shear waves arriving close to vertically from below, or by compressional waves traveling horizontally in the upper layer. The observed motion, though largely confined to the longitudinal, has an appreciable vertical component at short distances. The angle of this motion with respect to the horizontal is such as to favor the view that C consists of longitudinally polarized shear waves, as shown in figure 8.

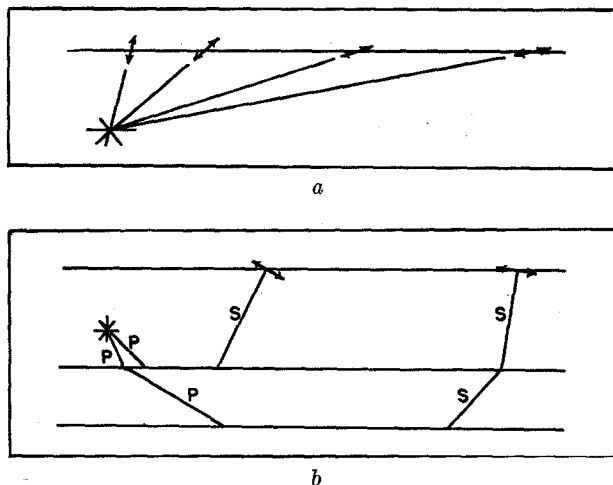


Fig. 8. *a.* Direction of particle motion of direct compressional ray arriving at the surface. Near the shot the motion is up-away and down-toward the shot location. At large distances the vertical component becomes very small. *b.* Direction of particle motion of refracted shear ray arriving at surface. Near the shot the motion is down-away and up-toward the shot location. The deeper it penetrates, the steeper is its arrival at the surface.

C often has the largest amplitude on the record, and since it also arrives during a longer interval of time than any other pulse it must carry a large part of the energy of the explosion. Although no shear wave would be expected to be generated by an underground explosion, at each interface both a reflected and a refracted shear wave will be produced (fig. 9), the relative amplitudes depending on the angles of incidence (see Gutenberg, 1944). At some angles the transverse refracted ray has the greatest amplitude. This is of particular significance near the surface where the wave velocities increase rapidly with depth, as then comparatively little energy will be transmitted downward. Only waves arriving at an interface at angles no larger than the critical angle,  $i_c$ , will enter the lower medium. The angle  $i_c$  is small if the velocity in the second medium greatly exceeds that in the first. Between the

critical angles for compressional and shear waves there will be a refracted transverse ray only; at larger angles, no refracted waves. Furthermore, if a free surface lies not far above the interface, when  $i_c$  lies between these two critical angles, the reflected P will soon again reach the interface after reflection from the free surface, and at such an angle that more of its energy will be fed into the second layer as refracted shear waves (fig. 10). These later gener-

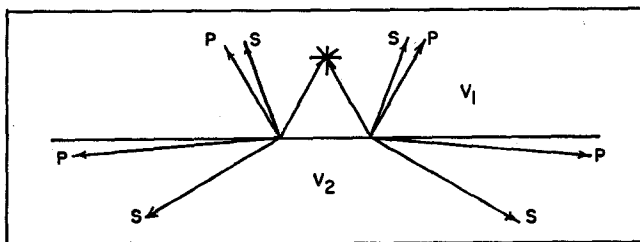


Fig. 9. Division of incident compressional rays at an interface when  $V_2$  is greater than  $V_1$ .

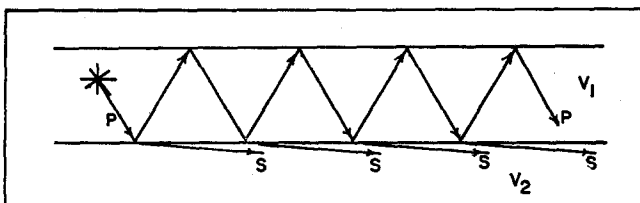


Fig. 10. Loss of energy from a compressional ray confined to a superficial layer by refraction as shear rays in the underlying material.

ated shear waves will not arrive at a distant point simultaneously with the first refracted waves, but will result in a series of pulses of shear waves, or a long continuous sequence of shear waves. This is very similar to the motion of C.

Body waves may also be dispersed by absorption, as given by the formula:

$$a = \frac{1}{\Delta_2 - \Delta_1} \log_e \frac{A_1 \Delta_1}{A_2 \Delta_2} \quad (3)$$

$a$  would be about .00073 per meter in this case. The wave length of C is of the order of 90 meters. Therefore, for body waves  $a$  would be about .066 per wave length; for surface waves, about .11 per wave length. In either case this is a very large absorption, though it is not necessarily enough to account for the dispersion.

No strongly dispersed body waves are found in the records of teleseisms, or even in those of explosions where the energy recorded has penetrated more than a few hundred feet into the ground. However, the work of Lampson (1946) and his co-workers has shown that the nonlinear relation of stress to strain in the ground at shallow depths will cause great distortion of the shape

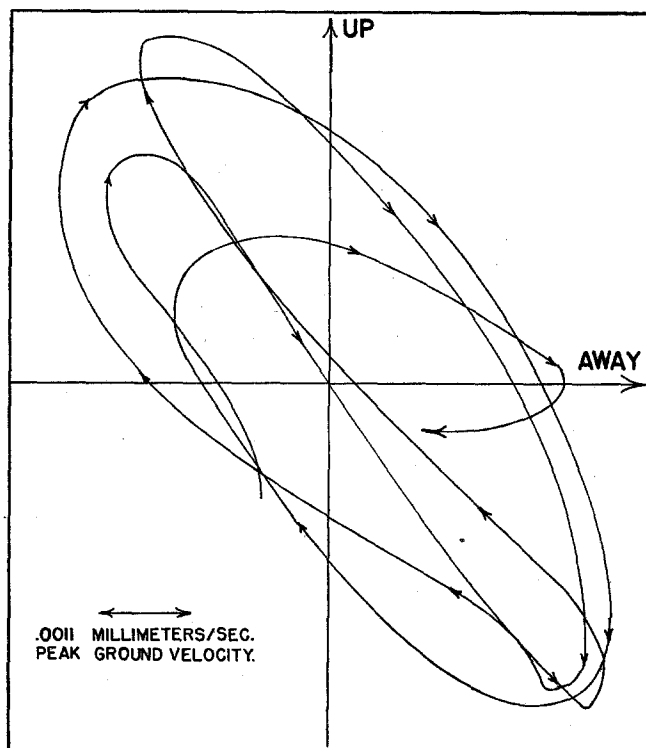


Fig. 11. Particle motion during the passage of H 1,493 meters from an explosion.

of the transmitted pulse, possibly even as great as that observed. Modification of our conception of wave transmission to take into account the effects of the special properties of the surface materials is needed before it will be possible to describe properly such a ground motion as C.

#### THE H PULSE

Starting shortly before the end of C another pulse can be recognized by phase-related motion on both the longitudinal and vertical components. The maximum of this pulse is labeled "H" on the records. The amplitude gradually increases to a maximum, then slowly dies out again. The beginning appears to

be shortly before the time of the maximum amplitude of C. Figure 11 is a plot of the particle velocity during the later part of the passage of H. During its arrival, particles of the ground move in elliptical paths. The motion viewed

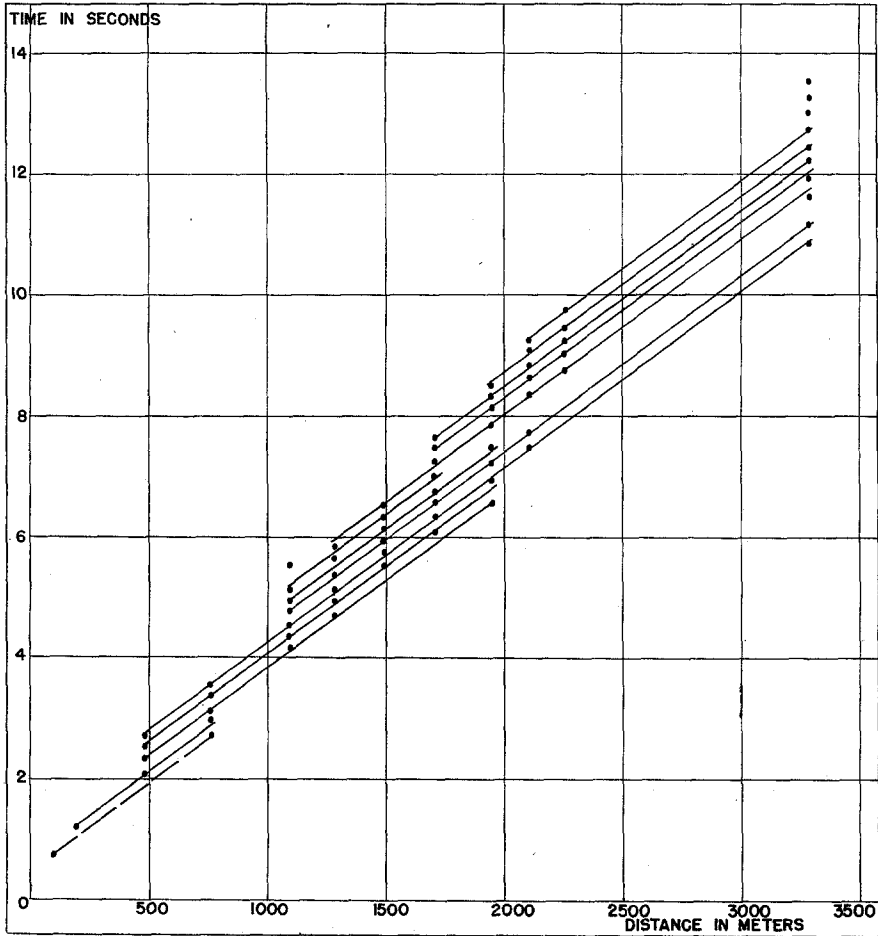


Fig. 12. Arrival times of H pulses.

from the surface is direct, not retrograde as in a Rayleigh wave. The horizontal component of motion has about twice the amplitude of the vertical. The ellipses are tilted with their tops toward the shot at an angle of about  $45^\circ$ . The average period of the waves is about .22 second. H is followed by a relatively quiet period during which the motion of the ground is irregular but appears to be generally similar to H. This may all be a part of the dispersed H pulse, or there may be several pulses of this type.

In figure 12 are plotted the times of arrival of the outstanding maximum downward velocities. Their group velocity is 345 m/s. The average velocity of the pencil of motions is 270 m/s.

H is similar to the hydrodynamic waves of Leet (1946), which also occur in about the same position with respect to other arrivals as H. However, the observed waves do not conform to the motion predicted by the classical gravity wave theory. The velocity,  $V_g$ , of gravity waves is given by the equation (see for instance Lamb, 1932, pp. 363 ff.):

$$V_g = \frac{g \times \text{period}}{2\pi} \tanh \frac{2\pi h}{\text{wave length}} \quad (4)$$

where  $g$  is the acceleration of gravity and  $h$  the thickness of the layer of fluid on the surface of which the waves are traveling. From the records we find that the period of H is of the order of .22 second; therefore, if H is some sort of gravity wave, and if the wave length is much less than  $h$ ,  $V_g = \frac{9.80 \times .22}{2}$  m/s.

= .34 m/s. (compared with the observed 270 m/s.). If the wave length is not small compared to  $h$ ,  $V_g$  will be even smaller. The large differences between these two velocities would have to be explained by the conditions of the ground. To do this, the present theory would have to be radically extended. At present there seems to be no relation between it and the observed waves except for the similarity of the particle paths.

On the other hand, H might be some form of Rayleigh wave wherein the direction of particle motion around the ellipses has been reversed from what is usual. Stoneley waves can be of this nature (Stoneley, 1924). The condition which must be satisfied if such waves are to exist is:

$$\begin{aligned} V_H^4 \{ (\rho_1 - \rho_2)^2 - (\rho_1 A_2 + \rho_2 A_1) (\rho_1 B_2 + \rho_2 B_1) \} \\ + 2KV_H^2 \{ \rho_1 A_2 B_2 - \rho_2 A_1 B_1 - \rho_1 + \rho_2 \} \\ - K^2 (A_1 B_1 - 1) (A_2 B_2 - 1) = 0 \end{aligned} \quad (5)$$

where

$$A_n = \left( 1 - \frac{V_H^2}{V_{Pn}^2} \right)^{\frac{1}{2}} \quad (6)$$

$$B_n = \left( 1 - \frac{V_H^2}{V_{Sn}^2} \right)^{\frac{1}{2}} \quad (7)$$

$$K = 2(\rho_1 V_{S1}^2 - \rho_2 V_{S2}^2) \quad (8)$$

$V_H$  is the velocity of Stoneley waves,  $V_{Pn}$  is the velocity of compressional waves,  $V_{Sn}$  that of shear waves, and  $\rho_n$  the density of the  $n$ th medium. The



direction of particle motion around the ellipses depends on the velocities of body waves in the two media. It can be either retrograde or direct. If the interface is viewed from medium 1 as a train of Stoneley waves passes, as shown in figure 13, and the motion appears to be retrograde, then if it is viewed from medium 2, as may be done simply by turning the page upside down, the motion will appear to be direct.

Too little is known about the thickness and elastic constants of the first layers at Los Alamos to determine if solutions of (5) are possible without

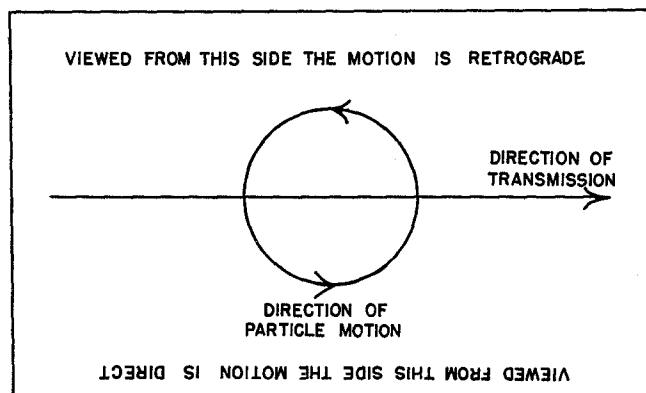


Fig. 13. Particle motion during the passage of a Stoneley wave.

making additional simplifying assumptions. Since except in the weathered layer the velocity of compressional waves greatly exceeds that of H,  $A_1$  and  $A_2$  differ very little from 1. If we assume that  $A_1 = A_2 = 1$ , and also that  $\rho_1 = \rho_2 = \rho$ , if neither  $B_1$  nor  $B_2$  equals 1, (5) reduces to:

$$-(B_2^2 - 1)(B_1^2 - 1)(B_2 + 1)(B_1 + 1) - 2(B_1 - B_2)^2(B_2 + 1)(B_1 + 1) + 2(B_2 + B_1)(B_1 - B_2)^2 = 0 \quad (9)$$

For real possible values of  $B_1$  there are no real possible values of  $B_2$  which satisfy this equation. Therefore, under the given conditions, Stoneley waves cannot exist.

Since these conditions are approximately what we would expect at any interface within the sedimentary section except the first one, it is unlikely that Stoneley waves with the velocity of the observed H waves can exist at Los Alamos, unless they are related to the interface at the base of the weathered layer. In this case, where  $V_H$  is 270 m/s., and  $V_{P2}$  is 1565 m/s.,  $A_2$  is 970 m/s. It will be seen later that the average value of Poisson's ratio,  $\sigma$ , is unusually large in the near-surface material, being probably close to .49. If we assume  $\sigma = .485$  in the first layer beneath the weathered layer,  $V_{S2} = V_H$

and  $B_2 = 0$ . If we also assume as before that  $\rho_1 = \rho_2 = \rho$ , equation (5) can be solved for  $A_1$ , giving:

$$A_1 = \frac{4 \left( 1 - \frac{V_H^2}{V_{S1}^2} \right)^{3/2} - .970 \frac{V_H^4}{V_{S1}^4}}{\left( 2 - \frac{V_H^2}{V_{S1}^2} \right)^2} \quad (10)$$

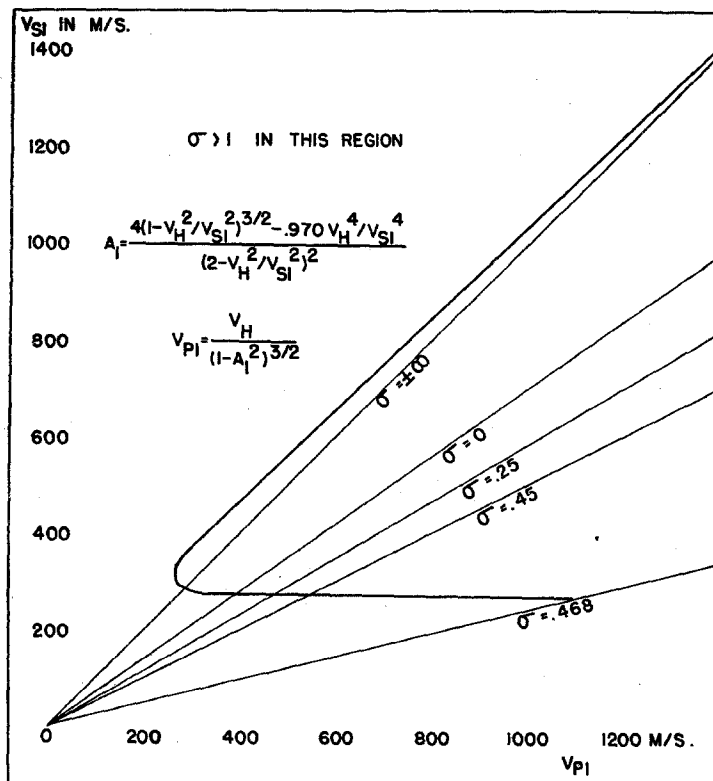


Fig. 14. Values of  $V_{P1}$  and  $V_{S1}$  which will satisfy Stoneley's equation if  $\rho_1 = \rho_2$  and  $V_{S1} = V_H$ .

If  $V_H$  exceeds  $V_{S1}$ ,  $A_1$  is complex. Therefore, we need consider only the range  $V_{S1} \geq V_H$ . From (6) and (10) we calculate  $V_{P1}$  for all values of  $V_{S1} \geq V_H$ . The resulting relation between  $V_{P1}$  and  $V_{S1}$  is shown in figure 14.

Not all of the solutions are physically possible. To determine which are possible we use Poisson's ratio:

$$\sigma = \frac{2 - \left( \frac{V_P}{V_S} \right)^2}{2 - 2 \left( \frac{V_P}{V_S} \right)^2} \quad (11)$$

Equation (11) defines a family of lines through the origin the directions of which depend on the values of  $\sigma$ . A number of these are shown in figure 14. Only one of the roots of  $V_{P1}$  corresponds to a physically possible value of  $\sigma$ , as values greater than  $\frac{1}{2}$  are impossible, and negative values have never been observed for real substances. It was stated that  $V_{P1}$  probably lay between 120 m/s. and 735 m/s. If the larger figure is more nearly the true one, Stoneley waves may be possible in the interface beneath the weathered layer.

If they can exist, we must investigate whether the particle motion will be direct or retrograde. It can be shown that the ratio of the vertical to the horizontal motion in the upper medium is:

$$\frac{w_1}{u_1} = -i \left[ \left( 1 - \frac{V_H^2}{V_{P1}^2} \right)^{\frac{1}{2}} \left( 1 - 2 \frac{V_{S1}^2}{V_H^2} \right) + 2 \frac{V_{S1}}{V_H} \left( \frac{V_{S1}^2}{V_H^2} - 1 \right)^{\frac{1}{2}} \right] \quad (12)$$

If  $V_{P1}$  lies between 395 m/s. and 735 m/s., the physically acceptable part of figure 14 gives  $i.32 \leq w_1/u_1 \leq i.67$ . Since Stoneley's  $z$  axis is directed into the upper medium, this means that the particle motion is retrograde elliptical, the reverse of what was observed for H waves.

Actually we should examine  $w_1/u_1$  at the surface of the ground, not at the interface. Since the surface layer is probably less than a quarter wave length in thickness, the motion at the surface might be expected to be similar to that at the interface.

Another possibility, not yet investigated thoroughly, is that H is the sum of a compressional and a shear pulse arriving from below at different angles. This approach seems to show promise, and will be made the subject of a later paper.

#### RAYLEIGH WAVES

The last observed pulse to arrive is recorded on the longitudinal and vertical components. Its maxima are labeled "R" on figures 3-5. Figure 15 is a plot of the particle velocity during its passage. The group velocity of its maximum is 193 m/s. (see fig. 6). The average velocity of the pulse is  $167\frac{1}{2}$  m/s.

R was frequently felt at the recording locations, and is a part of the "ground roll," which also may include C. The latter pulse, however, does not give the characteristic rolling sensation. H is generally smaller in amplitude, and not easily recognized. It is well known that the ground roll decreases with the depth of the shot. Therefore, theoretically and by analogy with tectonic earthquakes, where the relative size of the surface waves compared to the body waves is less the greater the depth of focus of the earthquake, this indicates that R is a surface wave. The retrograde, elliptical pattern of the motion suggests that it is a Rayleigh wave (Rayleigh, 1885). It is notable that the ellipses are tilted at about  $45^\circ$  with their tops away from the shots. The horizontal amplitude of motion never exceeds the vertical. Their ratio seems to

decrease gradually with distance where R can be clearly separated from earlier pulses. The average period of the motion in general increases with distance from the shot, but decreases with time of arrival at a given distance. The predominant wave length appears to vary from 33 to 46 meters, increasing with distance. However, because the seismograph is insensitive to frequencies below  $3\frac{1}{2}$  c.p.s., motion with longer wave lengths and periods would not be well

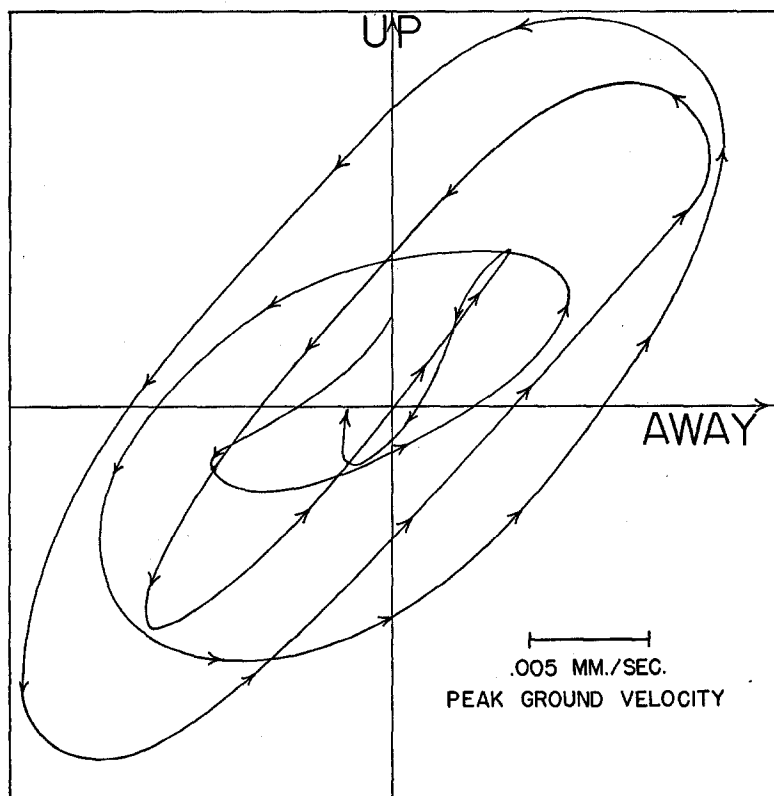


Fig. 15. Ground velocity during the arrival of R.

recorded. From equation (2) it can be shown that the absorption coefficient,  $\alpha$ , is about .0036 per meter, or .14 per wave length.

A large number of investigators have examined the range of existence of Rayleigh waves under different conditions. Fu (1946) has given the equations for the general case when a layer of one material overlies a half space of another. Love (1911, pp. 165-177), Meissner (1922), Stoneley (1926), and Bateman (1938) have examined what happens for special kinds of surface layers. Stoneley (1934) and Pekeris (1935) have investigated the case of a half space where the elastic constants are a function of depth. Considerable work

has also been done by Uller (see Gutenberg, 1932, pp. 115-124), who has developed general equations for the existence of all kinds of surface waves. The results of these investigations are so complicated that it has not been possible to correlate most of them with the observed data. The only one which will be treated here is the case of a homogeneous, isotropic, elastic half space overlain by a layer of fluid of thickness,  $h$  (Stoneley, 1926).

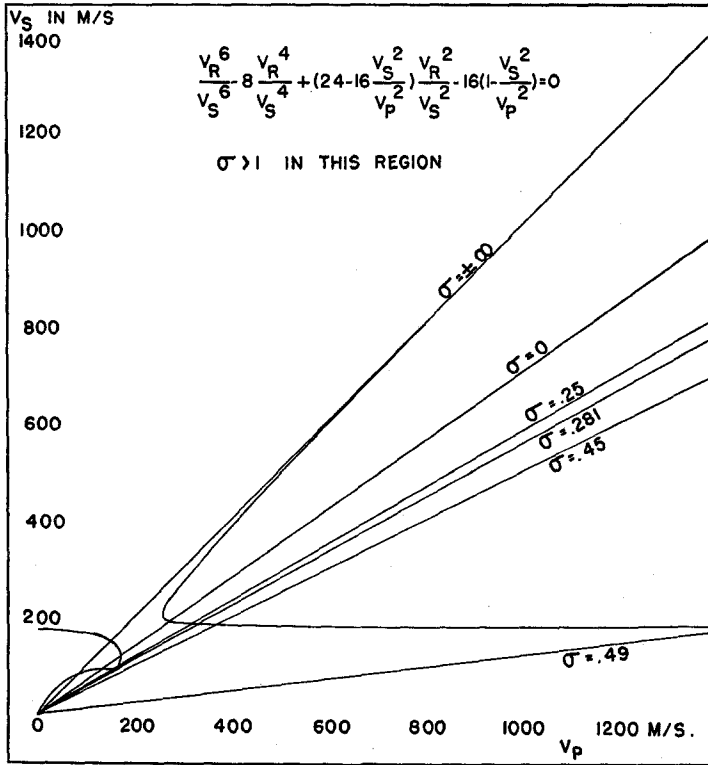


Fig. 16. Values of  $V_S$  and  $V_P$  which satisfy Rayleigh's equation if  $V_R = 167\frac{1}{2}$  meters per second.

The necessary condition for the existence of surface waves in this case is:

$$R(V_{P,R,S}) = \left(\frac{V_R}{V_S}\right)^4 \frac{\rho_1 \left(1 - \frac{V_R^2}{V_P^2}\right)^{\frac{1}{2}}}{\rho_2 \left(1 - \frac{V_R^2}{V_0^2}\right)^{\frac{1}{2}}} \tanh \left[ \frac{2\pi h}{L} \left(1 - \frac{V_R^2}{V_0^2}\right)^{\frac{1}{2}} \right] \quad (13)$$

where:

$$R(V_{P,R,S}) = 4 \left(1 - \frac{V_R^2}{V_S^2}\right)^{\frac{1}{2}} \left(1 - \frac{V_R^2}{V_P^2}\right)^{\frac{1}{2}} - \left(2 - \frac{V_R^2}{V_S^2}\right)^2 \quad (14)$$

$\rho_1$  and  $\rho_2$  are the densities of the fluid layer and the solid half space, respectively;  $V_0$ , the velocity of compressional waves in the fluid;  $V_P$  and  $V_S$ , the velocities of compressional and shear waves in the half space;  $V_R$ , the velocity of Rayleigh waves in the boundary between the two media; and  $L$ , the wave length. If  $V_R \ll V_S$ ,  $\rho_1 \ll \rho_2$ ,  $V_R \approx V_P$  or  $h \ll L$ , the right-hand side of this equation vanishes. What remains is one form of Rayleigh's equation. If we think of the air at Los Alamitos as being the fluid layer in this case, then  $\rho_1 \ll \rho_2$ , and  $V_R = 167\frac{1}{2}$  m/s. Figure 16 is a plot of the resulting relation between  $V_P$  and  $V_S$ . Since the wave length of R is about 40 meters, the surface waves must travel partly in the weathered layer and partly in the layer beneath it. Since the probably maximum thickness of the weathered layer is less than one-quarter of this wave length, the pulse must travel primarily in the deeper layer, the compressional wave velocity of which is 1565 m/s. The possible solution of (13) shown in figure 16 for this value of  $V_P$  corresponds to a Poisson's ratio of about .49.

In the ground at the interface beneath the fluid layer the ratio of the vertical to the horizontal amplitude of motion would be:

$$\frac{w}{u} = \frac{i}{2} \frac{R(V_{P,R,S}) + \left(2 - \frac{V_R^2}{V_S^2}\right)^2}{\left(1 - \frac{V_R^2}{V_S^2}\right)^{\frac{1}{4}} \left[2 - \frac{V_R^2}{V_S^2} - \frac{V_R^2}{V_S^2} R(V_{P,R,S})\right]} \quad (15)$$

( $w$  is positive up into the fluid,  $u$  is positive in the direction of propagation of the waves). Since  $R(V_{P,R,S}) = 0$  when  $\rho_1 \ll \rho_2$ , (15) reduces to:

$$\frac{w}{u} = i \frac{1 - \frac{1}{2} \frac{V_R^2}{V_S^2}}{\left(1 - \frac{V_R^2}{V_S^2}\right)^{\frac{1}{4}}} \quad (16)$$

Using the relation between  $V_P$  and  $V_S$  shown in figure 16, we find the relation between  $w/u$  and  $V_P$  as given in figure 17.

If  $V_R$  exceeds  $V_S$  the amplitude no longer decreases exponentially with depth, nor is the motion elliptical. Equation 16 becomes:

$$\frac{w}{u} = \pm \frac{1 - \frac{1}{2} \frac{V_R^2}{V_S^2}}{\left(\frac{V_R^2}{V_S^2} - 1\right)^{\frac{1}{4}}} \quad (17)$$

The corresponding solutions of Rayleigh's equation are body waves. The motion is, except in one special case, neither parallel nor perpendicular to the surface along which the waves are traveling, but is at some intermediate angle. It appears from equation (17) that  $w/u$  can have any value except  $\pm\infty$ ,

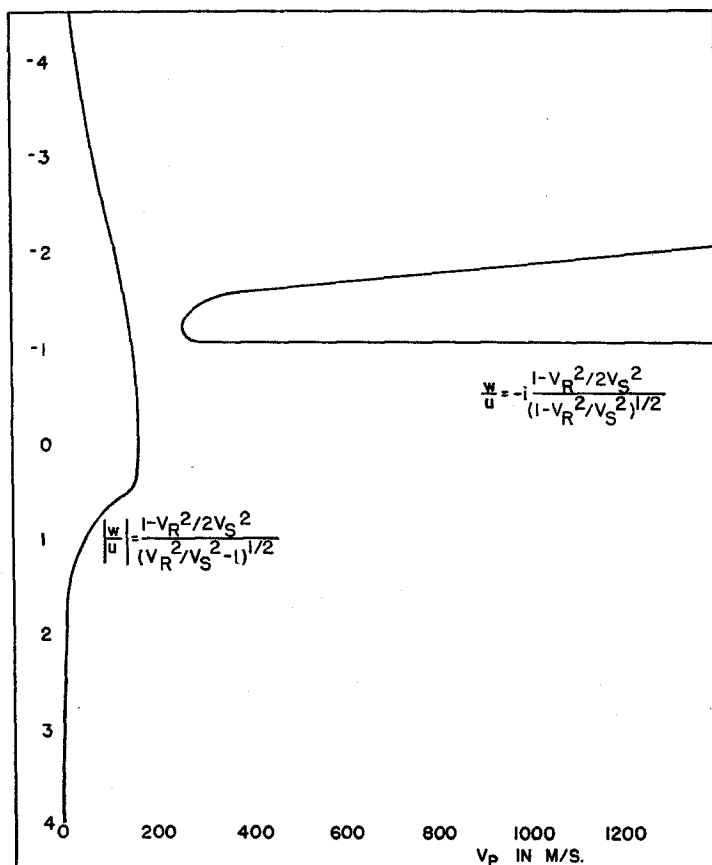


Fig. 17. Ratios of maximum vertical to horizontal amplitudes of Rayleigh waves when  $V_R = 167\frac{1}{2}$  meters per second.

which corresponds to the cases  $V_S = 0$  and  $V_S = V_R$  for which Rayleigh's equation is indeterminate or reduces to a contradiction. However, only a small range of these values corresponds to positive values of Poisson's ratio less than  $\frac{1}{2}$ . This is the range  $0 \leq |w/u| \leq .787$ . It can be shown that, except in the case  $V_P = V_R$ , the motion is the sum of a compressional and a shear wave of the same frequency arriving from different directions.

The case  $w/u = 0$  is of interest. Here  $(V_R/V_S)^2 = 2$ , and Rayleigh's equa-

tion reduces to  $(V_s/V_P)^2 = 1/2$ , which means Poisson's ratio is 0. This is the special case in which all roots of Rayleigh's equation are identical and  $V_P = V_R$ .

Figure 17 shows that if  $V_s > V_R$ ,  $w/u$  must exceed 1. As noted earlier, the observed  $w/u$  never exceeds 1. In the fluid layer the ratio of the vertical to the horizontal amplitude is:

$$\frac{w}{u} = i \left( 1 - \frac{V_R^2}{V_0^2} \right)^{\frac{1}{2}} \coth \left[ \frac{2\pi[h-z]}{L} \left( 1 - \frac{V_R^2}{V_0^2} \right)^{\frac{1}{2}} \right] \quad (18)$$

where  $z$  is the height above the bottom of the layer. Since for air  $V_0 = 344$  m/s. (Stewart and Lindsey, 1930, p. 238), and  $V_R = 167\frac{1}{2}$  m/s., at  $z = 0$ ;  $w/u = i.874 \coth 5.49 h/L$ . Unless  $h < L$ ,  $\coth 5.49 h/L \approx 1$ , and  $w/u \approx i.874$ . Therefore, the motion in the air is retrograde elliptical with the amplitude of the vertical component .874 times the amplitude of the horizontal. This calculated amplitude ratio is of the same order of magnitude as those observed. If  $h < L$ ,  $\coth 5.49 h/L > 1$ , and  $w/u$  becomes larger as the ratio  $h/L$  decreases. If the air acting in the manner described by Stoneley causes the observed low values of  $w/u$ , then the material very near the surface must behave as if it were a part of that medium instead of a part of the ground. This may explain why we observe only relatively low-frequency Rayleigh waves. The transition from air to ground may be so gradual that it looks like a sharp boundary only to vibratory movements the wave lengths of which exceed a certain value. Shorter waves might not be transmitted by such an interface at all.

It might be asked, next, if the weathered layer in the Los Alamitos area can be treated as a "fluid" layer by itself. In this case, since our observations would have been made at  $z = h$ , equation (18) reduces to  $w/u = \infty$ , which is not the case.

#### MOTION ON THE TRANSVERSE COMPONENT

Some transversely polarized energy is arriving throughout the records. It is possible that Love waves or transversely polarized body shear waves were recorded. The time of arrival of the first recognizable pulse of low-frequency energy in the transverse direction is marked "T" on the records. The motion in the transverse direction is so irregular, and the arrival times scatter so badly when plotted, that they have not been shown on the travel-time charts.

#### SUMMARY

In summary, the following pulses were recognized on many or all of the records taken in the Los Alamitos region:

- |                |  |
|----------------|--|
| P              | The first pulse to arrive.                   |
| P <sub>2</sub> | A compressional pulse arriving later than P. |



- $X_1, X_2, X_3$  Three pulses assumed to be body waves traveling along deeper paths than P and  $P_3$ .
- C A strongly dispersed pulse largely confined to the longitudinal component.
- T Motion on the transverse component arriving largely coincident with C.
- H A direct elliptical motion in a vertical plane.
- R A Rayleigh-type motion.

P, C, H, and R are prominent on all the records, though at distances less than 300 meters from the explosions C, H and R overlap each other, and are not separable. They are strongly dispersed, and their beginnings are therefore difficult to identify. It is the times of arrival of their maxima which are plotted in the travel-time curves.

The recorded pulses of energy are all of types previously reported. However, no completely satisfactory theory explaining C, H, or R exists. Any theory describing these pulses must be based on a knowledge of the fundamental properties of the first few meters of the earth's crust, a complicated medium which is plainly neither homogeneous nor elastic. The fact that such a medium can transmit both compressional and Rayleigh-type waves is an encouraging sign, since it means that its behavior can not be radically different from that of elastic substances. It is to be hoped that the mathematical physicists will soon develop equations describing wave transmission through such media.

In closing, the author would like to thank the many persons, especially Dr. Beno Gutenberg of the California Institute of Technology and Dr. R. A. Peterson of United Geophysical Company, for their advice and assistance, without which this research could not have been done.

#### BIBLIOGRAPHY

BATEMAN, H.

1938. "Rayleigh Waves," *Proc. Nat. Acad. Sci.*, 24: 315-320.

FU, C. Y.

1946. "Rayleigh Waves in a Superficial Layer," *Geophysics*, 11: 10-23.  
 1947. "Propagation of Elastic Waves in the Neighborhood of a Free Boundary," *Geophysics*, 12: 57-71.

GUTENBERG, B.

1932. *Handbuch der Geophysik*, Vol. 4. (Berlin, Gebrüder Borntraeger).  
 1944. "Energy Ratio of Reflected and Refracted Seismic Waves," *Bull. Seism. Soc. Am.*, 34: 85-102.

JEFFREYS, H.

1925. "On the Surface Waves of Earthquakes," *Mon. Not. Roy. Astron. Soc., Geophys. Suppl.*, 1: 282-291.

LAMB, H.

1904. "On the Propagation of Tremors over the Surface of an Elastic Solid," *Phil. Trans. Roy. Soc. London*, 20: 1-42.  
 1932. *Hydrodynamics*, chap. 9, pp. 363-475 (Cambridge Univ. Press).

## LAMPSON, C. W.

1942. "Preliminary Measurements of Earth Pressures from the Explosion of Small Charges of T. N. T.," Natl. Res. Coun. Comm. on Passive Protection against Bombing, interim report no. 16.
1946. "Final Report on Effects of Underground Explosions," Natl. Def. Res. Comm., report A-479.

## LEET, L. D.

1939. "Ground Vibrations near Dynamite Blasts," *Bull. Seism. Soc. Am.*, 29: 487-496.
- 1946a. "Earth Motion from the Atomic Bomb Test," *American Scientist*, 34: 198-211.
- 1946b. "Vibrations from Blasting," *Explosives Engineer*, 24: 41-44, 55-59, 85-89.

## LEET, L. D., and EWING

1932. "Velocity of Elastic Waves in Granite," *Jour. Appl. Physics*, 2: 160-173.

## LOVE, A. E. H.

1911. *Some Problems of Geodynamics* (Cambridge Univ. Press).

## MEISSNER, E.

1922. "Oberflächenwellen bei Mitschwingen einer tragen Rindenschicht," *Vierteljahrs. Naturf. Ges. Zurich*, Vol. 67, p. 1. See also Gutenberg, 1932, *Handbuch der Geophysik*, pp. 94-97.

## PEKERIS, C. L.

1935. "The Propagation of Rayleigh Waves in Heterogeneous Media," *Jour. Appl. Physics*, 6: 133-138.

## RAYLEIGH, LORD

1885. "On Waves Propagated along the Plane Surface of an Elastic Solid," *Proc. London Math. Soc.*, 17: 4-11. Also *Coll. Works* 2: 441-447.

## RUST AND MOUNCE

1942. "Measurements of Earth Pressures under Detonation," Natl. Res. Council Comm. on Passive Protection against Bombing, interim report no. 19.

## SHARPE, J. A.

1942. "The Production of Elastic Waves by Explosion Pressures," *Geophysics* 7: 144-154, 311-322.

## STEWART AND LINDSEY

1930. *Acoustics* (New York, Van Nostrand).

## STONELEY, R.

1924. "Elastic Waves at the Surface of Separation of Two Solids," *Proc. Roy. Soc.*, ser. A, 106: 416-428.
1925. "Dispersion of Seismic Waves," *Mon. Not. Roy. Astron. Soc., Geophys. Suppl.*, 1: 280-282.
1926. "The Effect of the Ocean on Rayleigh Waves," *Mon. Not. Roy. Astron. Soc., Geophys. Suppl.*, 1: 349-356.
1934. "The Transmission of Rayleigh Waves in a Heterogeneous Medium," *Mon. Not. Roy. Astron. Soc., Geophys. Suppl.*, 3: 222-232.

## WEATHERBY, B. B.

1943. "Effects of Subsurface Detonations in Earth II," Natl. Def. Res. Comm., Armor and Ordnance Report A-238.